Cylindrical and spherical pistons as drivers of MHD shocks

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Introduction

Coronal MHD shock wave signatures (type II burst and Moreton wave) appear in close association with the flare impulsive phase and/or the CME acceleration phase. A necessary requirement for the shock formation is a motion perpendicular to the magnetic field lines, i.e., the source-region expansion generally might be caused by a flare or CME. In any case, the expansion creates a large-amplitude perturbation in the ambient plasma. The shock is formed after certain time/distance due to the non-linear evolution of the perturbation wavefront (larger-amplitude elements propagate faster; [1]).

We investigated a general situation (regardless of the CME/flare nature of the source-region expansion), employing the cylindrical and spherical symmetry (2D and 3D piston, respectively). We considered an accelerated motion of the piston surface which creates a perturbation and determines the evolution of the wavefront profile. In particular, high b (≈ sound) and low b cases have been analyzed.

Model

The source-surface speed, $v(t)$, at a certain time t is defined by the initial velocity v_0, the final velocity v_{2max}, the acceleration time t_2. Due to energy conservation, the signal amplitude has to decrease with increasing distance, which is basically different from the 1D model [2]. For example, in the case of $b >> 1$, where only the kinetic energy has to be taken into account, we use:

$$\underbrace{r\,u^2 w R^a}_{\text{energy flux}} = const. \Rightarrow g(u)s^a = const. \qquad (1)$$

(a = 1 for cylindrical; a = 2 for spherical coord. system)

Generally, $g(u)$ depends on the characteristics of the ambient plasma, primarily on the value of b.

The basic characteristic of the non-linear wavefront evolution is that the wave speed, w, depends on the wave amplitude, u:

$$w(t) = w_0 + k\,u(t), \qquad (2)$$

where u is the flow speed. The parameter k and speed w_0 depend on b; in the $b << 1$ case: $k = 3/2$, $w = v_{A0}$ (Alfvén speed) [1] and $b >> 1$: $k = 4/3$, $w = c_{s0}$ (sound speed) [2].

Equations (1) and (2) give differential equations for the flow speed $u(t)$. In the cylindrical and spherical coordinate system the equations governing the wave propagation can be expressed as:

$$-r(t_0)g(u_0)\frac{1}{g^2(u)}\frac{\mathrm{d}g(u)}{\mathrm{d}t}\frac{\mathrm{d}u(t)}{\mathrm{d}t} = w_0 + ku, \qquad (3)$$

$$-\frac{1}{2}r(t_0)\sqrt{g(u_0)}\left[g(u)\right]^{-3/2}\frac{\mathrm{d}g(u)}{\mathrm{d}t}\frac{\mathrm{d}u(t)}{\mathrm{d}t} = w_0 + ku, \qquad (4)$$

with $u_0 \equiv u(t_0) = v\,(t_0)$ where $u(t_0)$ is the source velocity at the moment t_0 when a given wave segment was created.

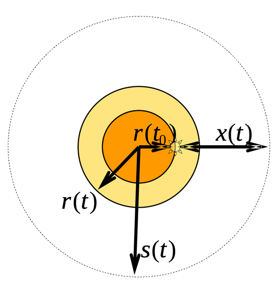

Figure 1. 3D (2D) piston signal propagation

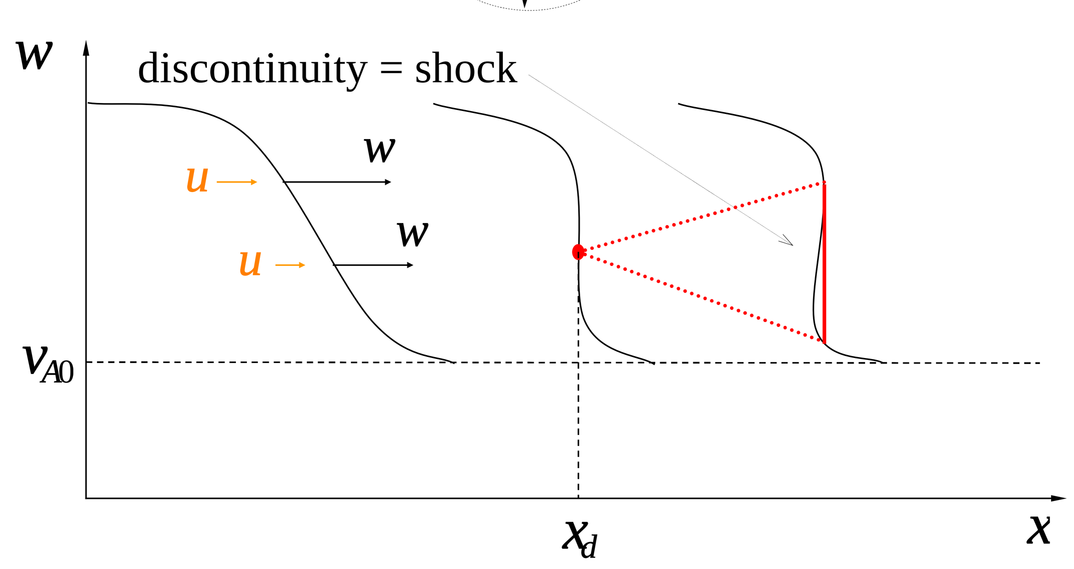

Figure 2. Non-linear wavefront evolution and shock formation at x_d.

Results

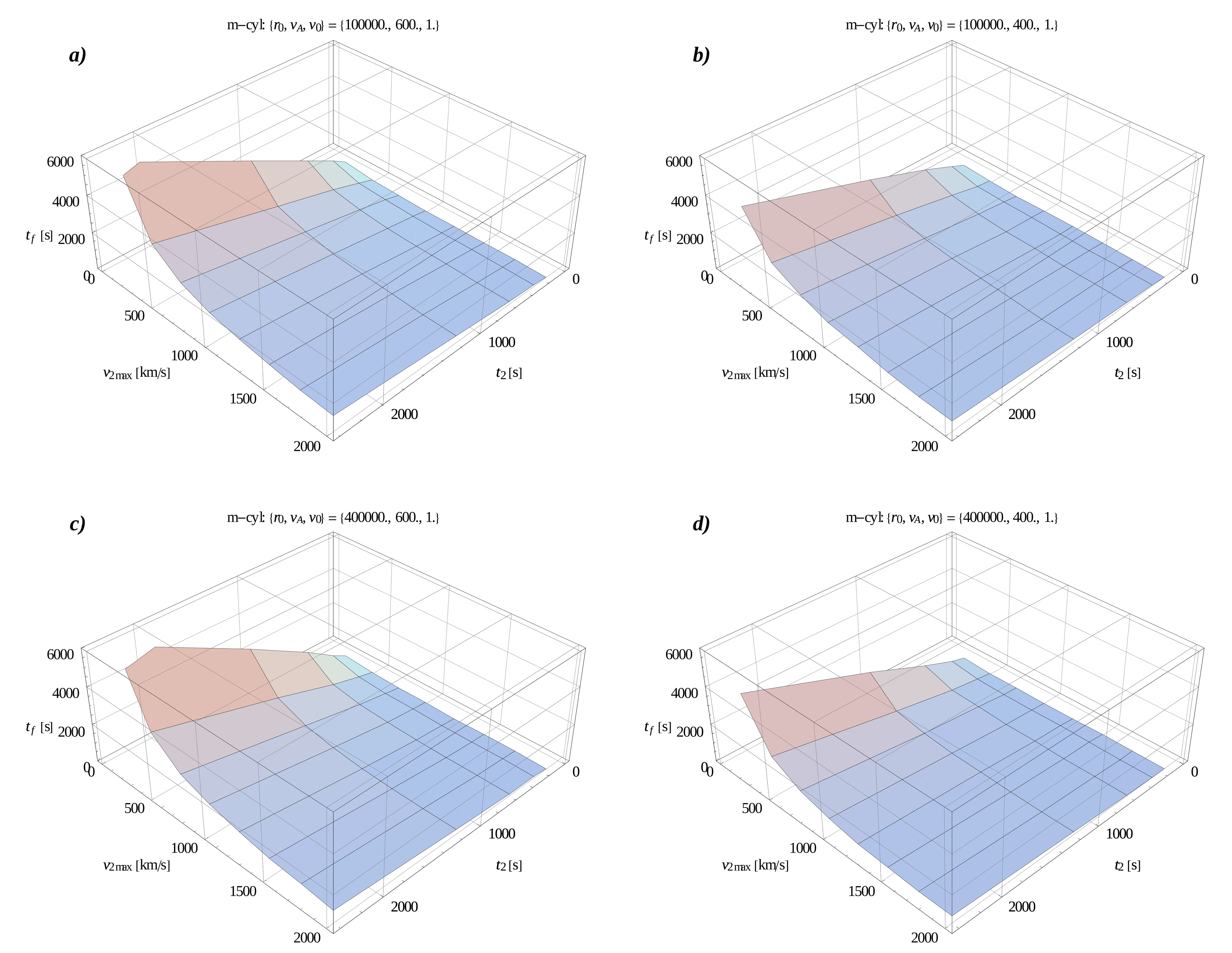

Figure 4. The MHD shock-formation time t_f dependence on the acceleration time t_2 and the maximum speed of the expanding source-surface v_{2max} for the four cases of different initial size of the source-surface, r_0, and different Alfvén speeds, v_A. It can be seen that the time t_f weakly depends on r_0 and also that increases with increasing of v_A. The figures show dependences only in the cylindrical symmetry because the spherical case gives (qualitatively) similar results.

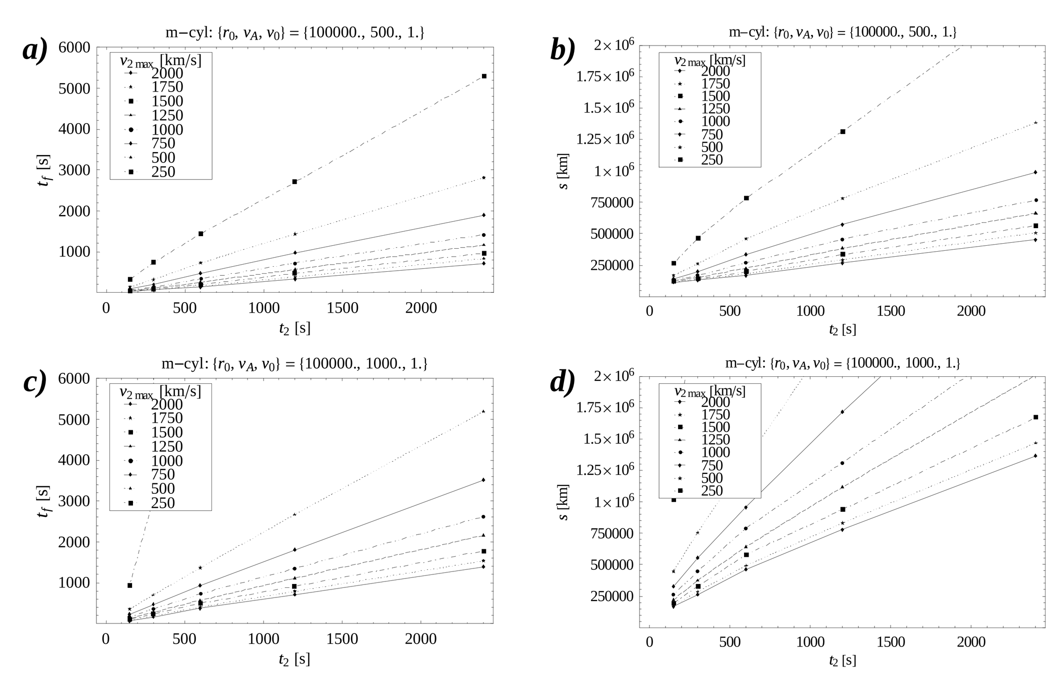

Figure 5. The left set of figures (a, c) represents the MHD shock-formation time, t_f, dependence on acceleration time, t_2, where the right (b, d), the shock-formation distance, s, dependence of the same acceleration time. In every figure, the mentioned dependences for different maximal source-surface speed, v_{2max}, are shown. All of them give dependences for the same initial source-surface diameter, i.e. r_0 = 100 Mm. The upper figure set (a, b) differs from the lower (c, d) in a different value for Alfvén speed v_A: for the upper set v_A = 500 km/s and for the lower v_A = 1000 km/s. These figures illustrate the cross-section of 3D $t_f(t_2,v_{2max})$ plots shown in Fig. 4.

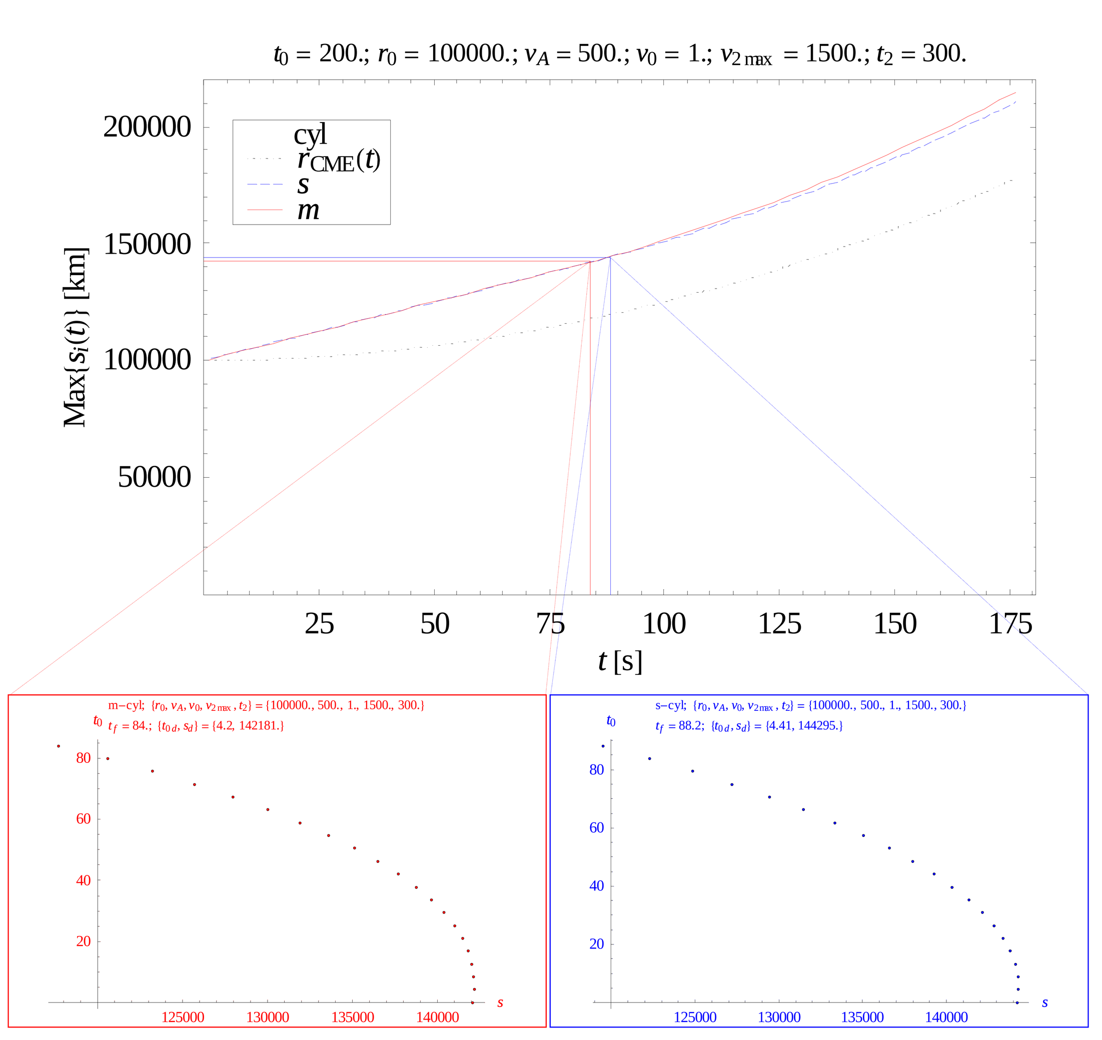

Figure 3. The upper, larger figure depicts the wavefront propagation in time for two different kinds of the wave: (s) sound- and (m) magnetohydrodinamic-like in a cylinder symmetry. The two lower figures give the wave distance-segment time creation profiles at the shock-forming time, t_f for these two kinds of waves.

Conclusion

We investigated dependences of the shock-formation times and distances on: the acceleration phase duration, t_2, the maximum expansion velocity (defining also acceleration), v_{2max}, the Alfvén velocity (defining also Mach number), v_A, the initial size of the piston, r_0. It has been shown that the MHD shock-formation time, t_f (Figs. 4 and 5):

- § is proportional to t_2
- § decreases with increasing v_{2max}
- § increases with increasing v_A
- § is weakly dependent on r_0,
- § $t_f \propto 1/a$.

In summary, the results explicitly show that the shock forms earlier for more impulsive acceleration.

Also the shock-formation distance, s_d (Fig. 5):

- § is proportional to t_2
- § decreases with increasing v_{2max}
- § increases with increasing v_A
- § increases with increasing of r_0

and is also dependent on the source-surface acceleration mode, i.e. $s_d \propto 1/a$.

Practically, it has been shown that the shock-formation time, t_f, is most directly related to the acceleration time of the source-region, t_2. If the source-region acceleration time is long enough, the shock does not need to form.

The most common misapprehension is that a shock only forms when the expansion is supersonic. In the case of 3D piston mechanism generally this is not true, i.e. to form a shock the supersonic expansion is not needed (well known fact [1]).

References

[1] Landau, L.D. and Lifshitz, E.M.: *Fluid Mechanics*, (Pergamon Press, 1987)

[2] Vršnak, B. and Lulić, S.: *Formation of Coronal MHD Shock Waves: I. The Basic Mechanism*, Solar Phys., **196** (2000) 157-180(24)